

Advanced Geometric Camera Calibration for Machine Vision

M. Vo, **Z. Wang**, L. Luu, and J. Ma, "[Advanced geometric camera calibration for machine vision](#)," *Optical Engineering*, Vol. 50, No. 11, 110503, 2011.

I. General description

In many machine vision applications, a crucial step is to accurately determine the relation between the image of the object and its physical dimension by performing a calibration process. Even though significant progress has been made, challenges remain due to the ever increasing demands for higher accuracy and performance. There are two main sources of error that affect the camera calibration results:

1. The assumption of a perfect planar target with ideal patterns of conventional camera calibration techniques. The defects of the calibration target lead to inaccurate results
2. The uncertainty in locating the control points directly using the geometries of the calibration patterns suffers from lens distortion as well as perspective distortion from raw images.

Therefore, it is crucial that the process must be carried out using expensive equipments under controlled illumination for conventional algorithms to yield accurate results. This project aimed at providing highly accurate camera parameters even with low cost hardware. The two-step approach is proposed to cope with the problem:

1. Bundle adjustment is employed to optimize the all the camera parameters and the defects of the calibration target.
2. The combination of the frontal image concept and digital image correlation is utilized, allowing hyper-precise control point detection in the image plane.

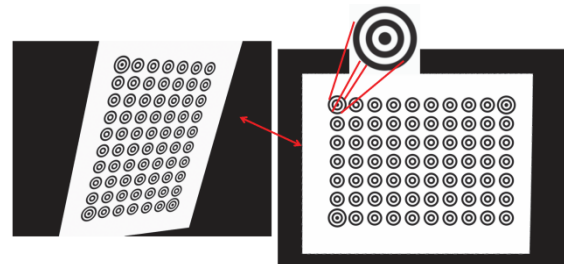
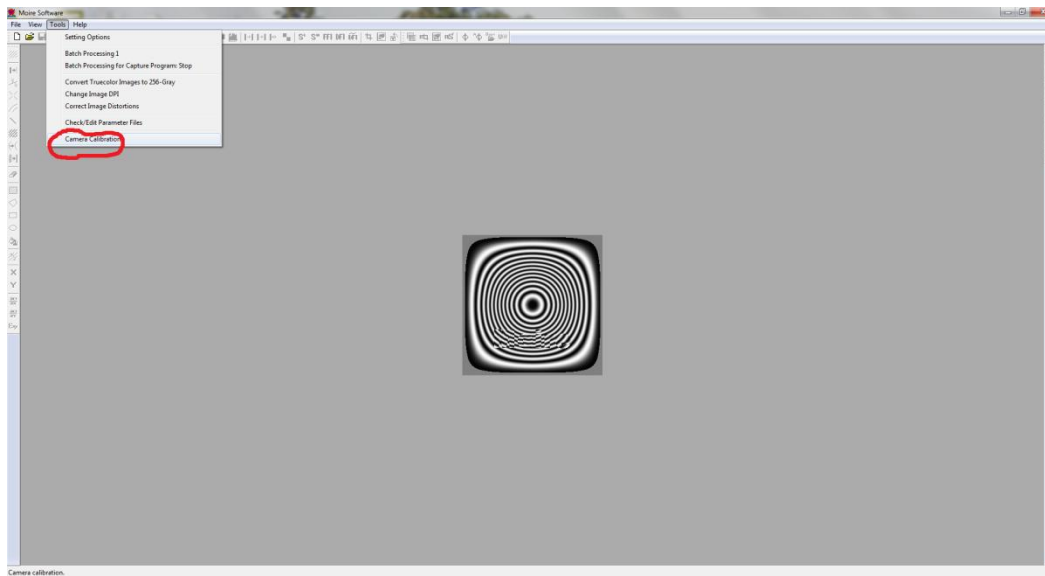


Figure 1: The conversion from raw images to frontal images enables hyper detection of the center of the concentric circles via Digital Image Correlation

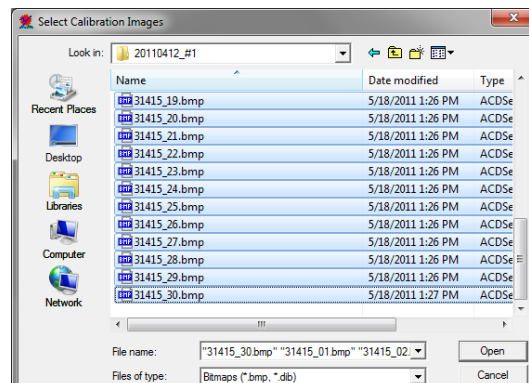
Note:

- The conventional methods refer to Zhengyou Zhang[1] and Jane Heikkilä [2] approaches. These two methods, especially Zhang's method, are very well-known and have been incorporated in the widely-used OpenCV package[3] and Matlab Camera Calibration toolbox[4].
- Users to this software package are required to cite this paper in any of their published work:
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II. Calibration example:



Step 1: Click on Tools tab to access Camera Calibration function



Step 2: Open calibration images.



Step 3: Selection calibration modes:

- Checker- Regular mode: offer reasonable accuracy in a relatively fast manner by employing Bundle Adjustment. The function works with checker board template.
- Checker-Advanced mode: offer the highest accuracy by employing advanced control point refinement and Bundle Adjustment. It is normal if it takes 2-3 minutes to complete the task. The function works with checker board template.
- Ring- Regular mode: offer reasonable accuracy in a relatively fast manner by employing Bundle Adjustment. The function works with ring board template.
- Ring-Advanced mode: offer the highest accuracy by employing advanced control point refinement and Bundle Adjustment. It is normal if it takes 3-4 minutes to complete the task. The function works with ring board template. **(Recommended)**
- Planar coded target- Regular: offer reasonable accuracy in a relatively fast manner by employing Bundle Adjustment. This function works with coded target template.
- Planar coded target- Advanced: offer the highest accuracy by employing advanced control point refinement and Bundle Adjustment. It is normal if it takes 5-6 minutes to complete the task. The function works with coded target template.
- Planar coded target- Regular: This function is still under testing.

Camera Calibration Parameters

Num. of control points per row:	10	Num of advanced refining (0-3):	1
Num. of control points per col:	7	Display refining details (0/X):	0
Number of images:	30	Refi. para (0-P6/1-P8/2-P8F):	0
Reserved:	0	Interpola. (0-6, 0-Fast/5-Accu):	5
Distance between ctrl points:	25.4	mm(in)	
Lens model (0-4: 18,16,12,10,8):	2	Ctrl-coeff for edge detect.<1.0):	0.87
Iter threshold:	0.001	(Low) Thresh. for edge detect.:	20
		(High) Thresh. for edge detect.:	100

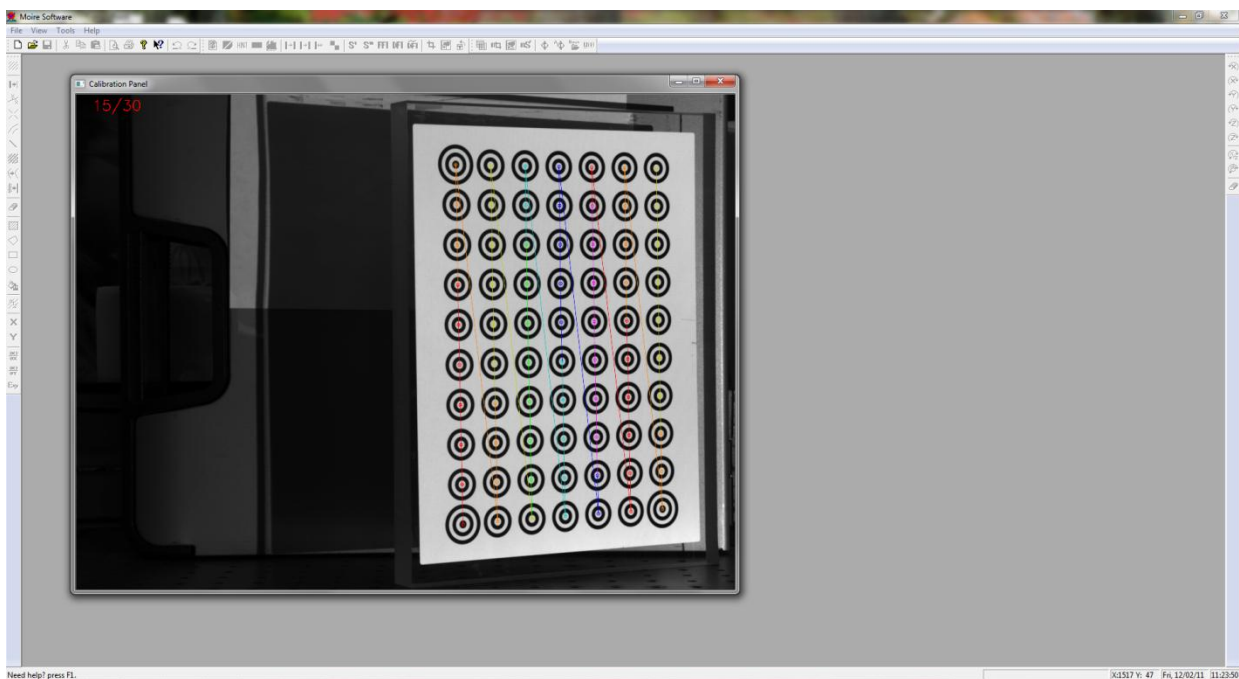
Instruction: The above parameters [if applicable] are required for Camera Calibration.

OK Cancel

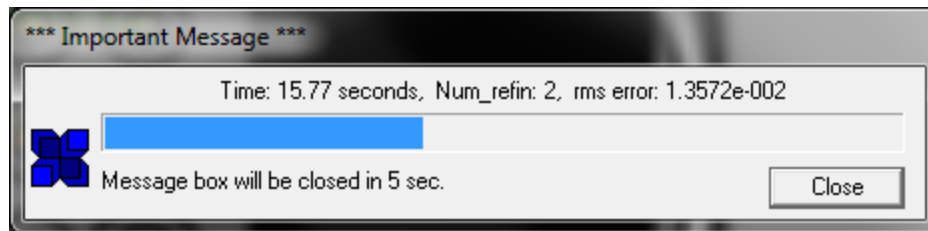
Step 4:

- Lens model.
 - Mode 0: Using full lens distortion model with 13 parameters
 - Mode 1: Using full radial, tangential and 2st order prism distortion model with 11 parameters

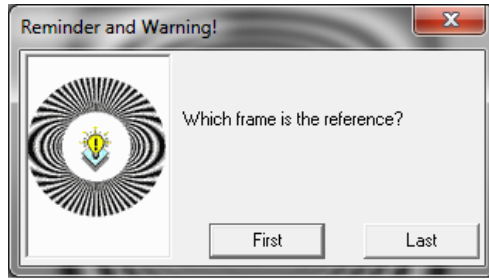
- Mode 2: Using 6th order radial, 2nd order tangential, and 2nd order prism distortion with 7 parameters (**Recommended**)
 - Mode 3: Using 6th order radial, 2nd order tangential distortion with 5 parameters (Similar to OpenCV)
 - Mode 4: Using 6th order radial distortion with 3 parameters.
→ Please refer to part III for further details of the distortion parameters
- Refi. Para: 0 in case of ring board and 1 for checker and coded board.
 - Inter. Para: Interpolation method used with the modes increase from fine to ultra fine interpolations. Please refer to reference 5 for further details about the interpolation methods.
 - [Low] and [High] Thres. for Edge Detect: Canny detection parameters.



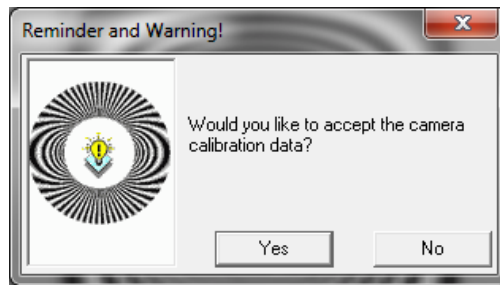
Step 5: control point extraction. This step is done automatically.



Step 6: The result of the calibration. The Root Mean Square (RMS) reprojection error is 0.013572 pixels



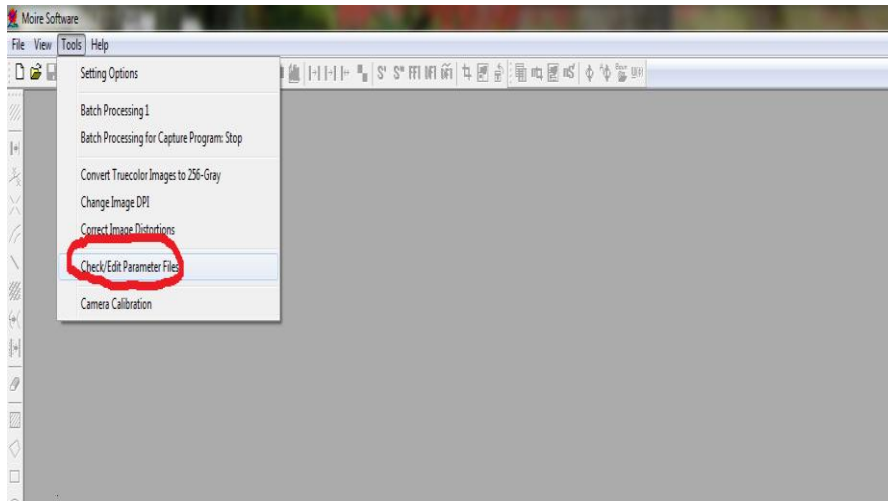
Step 7: The user selects whether the output will be based on either the first or the last image.



Step 8: Accept the camera calibration parameters.



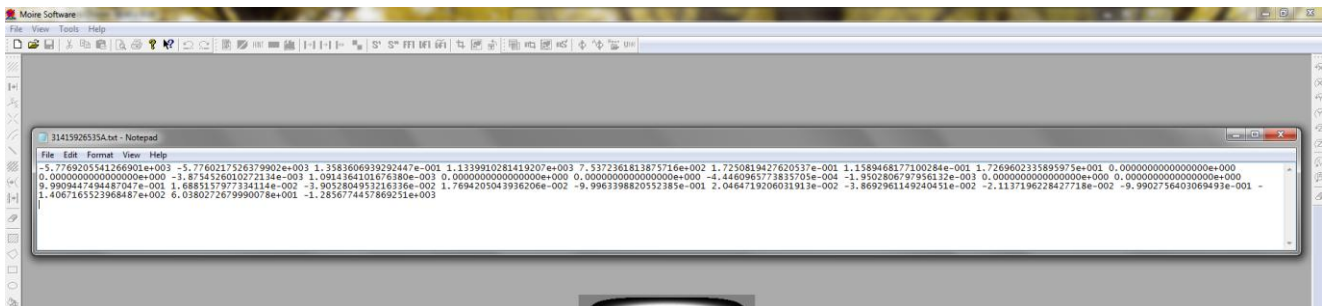
Step 9: the RMS reprojection error for each image and their mean error are shown.



Step 10: Click on the Tools tab to Check and Edit calibration parameters.



Step 11: Click on CC30 to Check and Edit calibration parameters.



Step 12: The calibration parameters are shown.

III. Description of the parameters:

- Physical world coordinate: (x_w, y_w, z_w)
- Image coordinate: (u, v)
- Focal length in pixel: (α, β)
- Skew factor: γ

- Principal point: (u_0, v_0)
- Distortion:
 - Radial distortion: a_0, a_1, a_2, a_3, a_4
 - Tangential distortion: p_0, p_1, p_2, p_3
 - Prism distortion: s_0, s_1, s_2, s_3
- Rotation matrix: $R_{11}, R_{12}, \dots, R_{33}$
- Translation matrix: T_1, T_2, T_3

Ideal pinhole lens model:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \sim \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} & R_{13} & T_1 \\ R_{21} & R_{22} & R_{23} & T_2 \\ R_{31} & R_{32} & R_{33} & T_3 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Lens distortion:

$$\begin{aligned} \tilde{x}_{cn} &= (p_0 + r^2 p_2)(r^2 + 2x_{cn}^2) + 2(p_1 + r^2 p_3)x_{cn}y_{cn} + (1 + a_0 r^2 + a_1 r^4 + a_2 r^6 + a_3 r^8 + a_4 r^{10})x_{cn} + s_0 r^2 + s_2 r^4 \\ \tilde{y}_{cn} &= (p_1 + r^2 p_3)(r^2 + 2y_{cn}^2) + 2(p_0 + r^2 p_2)x_{cn}y_{cn} + (1 + a_0 r^2 + a_1 r^4 + a_2 r^6 + a_3 r^8 + a_4 r^{10})y_{cn} + s_1 r^2 + s_3 r^4 \end{aligned}$$

With:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{cn} \\ y_{cn} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \tilde{u} \\ \tilde{v} \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_{cn} \\ \tilde{y}_{cn} \\ 1 \end{bmatrix}$$

$$r^2 = x_{cn}^2 + y_{cn}^2$$

Where:

(\tilde{u}, \tilde{v}) : distorted image point

(u, v) : distortion-free image point

An example output of the toolbox which can be found in file 31415926535A.txt as described in Step 11 and 12 above. The orders are as follow: $\alpha, \beta, \gamma, u_0, v_0, a_0, a_1, a_2, a_3, a_4, p_0, p_1, p_2, p_3, s_0, s_1, s_2, s_3, R_{11}, R_{12}, R_{13}, R_{21}, R_{22}, R_{23}, R_{31}, R_{32}, R_{33}, T_1, T_2, T_3$

Important convention:

- The origin of the image pixel coordinate is in lower left origin.
- The origin of the world coordinate is in lower left corner of the calibration board.

IV. References:

1. Z. Zhang, "A flexible new technique for camera calibration," *IEEE Trans. Pattern Anal. Mach. Intell.* Vol. 22, No. 11, pp. 1330–1334 (2000).
2. Heikkilä, J, "Geometric Camera Calibration Using Circular Control Points", *IEEE Trans. Pattern Anal. Mach. Intell.* Vol. 22, No. 10, pp. 1066-1077, Oct 2000.
3. <http://opencv.itseez.com/modules/calib3d/doc/calib3d.html>
4. http://www.vision.caltech.edu/bouguetj/calib_doc/htmls/links.html
5. L. Luu, Z. Wang, M. Vo, T. Hoang, and J. Ma, "Accuracy enhancement of digital image correlation with B-spline interpolation," *Optics Letters*, Vol. 36, No. 16, pp. 3070-3072, 2011.