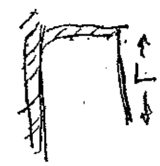
Preliminary Exam Physics Department 2008

You may use mathematical tables and non-programmable calculators provided by the exam proctor.

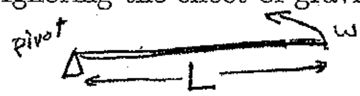
Classical Mechanics

1. Consider a rope of mass M (distributed uniformly along its length) and length L hanging from a tree-limb, as shown.



a) draw the force diagram for the rope.
b) What is the Tension, T, at the top of the rope?
c) What is T at point x, somewhere in the middle of the rope?
d) What is T at the bottom of the rope?

Now consider a whirling rope (same M and L as before), attached as shown. It is whirling with a uniform velocity, ω. Ignoring the effect of gravity:



e) What is T at a point r from the pivot point? Note: you will need to consider a small section of rope between r and $r + \Delta r$. Since the length of this segment is Δr, the mass in the segment is simply $\Delta m = M\Delta r/L$. Remember, since this is circular motion, the section experiences radial acceleration. Start, as usual, with a force diagram. Let the tension at the pivot point be T_0. You will need to explore the limit where $\Delta R \to 0$.

2. Consider a particle restricted to motion on the surface of a cone, as shown, such that $z = r cot(\alpha)$

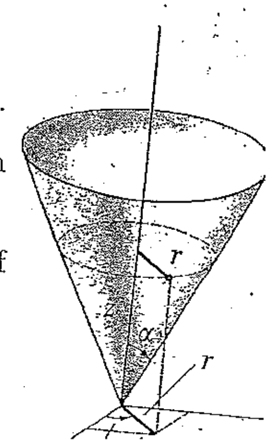
a) Noting that the velocity in cylindrical coordinates is of the form $v = dr/dt\mathbf{n_r} + rd\theta/dt\mathbf{n_\theta} + dz/dt\mathbf{n_z}$, where the n_i's are unit vectors in the $i = r, \theta, z$ direction respectively, determine v^2.

b) What is the potential energy, U, of the particle (assuming $U = 0$ at $z = 0$)?

c) Write the Lagrangian for the particle, using r and θ as the generalized coordinates.

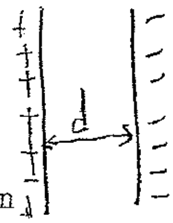
d) from the Lagrangian equation for the coordinate θ, show that angular momentum is conserved (i.e., its time derivative will be 0).

e) Now, using the Lagrangian equation for the coordinate r, obtain the equation of motion (in r).



Electricity & Magnetism

3. Consider a capacitor, with a separation of d:

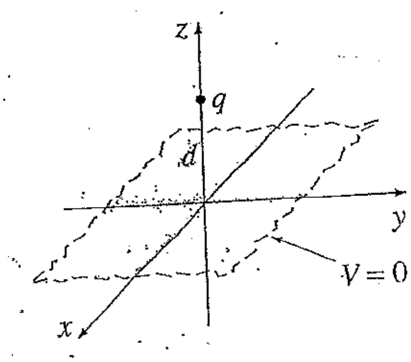


a) if the plates have surface charge $= \pm\sigma$, respectively, what is the $\mathbf{E}$ field between the plates (use Gauss's Law)?
b) What is the electric potential, V, between the plates?

Now assume the vacuum between the plates is filled with a dielectric of dielectric constant K.
c) What is the Electric field within the dielectric.
d) What is the induced surface charge on the dielectric adjacent to the positively charged plate.

4. This problem uses the image charge method. Consider a point charge, q, a distance d from an infinite grounded conducting plan.

a) determine the position and charge of the image charge.
b) determine the potential, V, in the region above the plane (use Cartesian coordinates). Recall what V is at the surface of the plane.
c) using your answer to part (a), what is the induced surface charge, σ? You will get partial credit if you are able to write the correct relationship between σ and V.
d) what is the Force on charge q?



Thermodynamics

5. a) The best laboratory vacuum has a pressure of about 1.00×10^{-18} atm, or 1.01×10^{-13}Pa. How many [redacted] moles of gas are there per cubic centimeter in such a vacuum at a Temperature of 293K?

b) Express your answer to part (a) in number of molecules per cubic centimeter.

c) How many atoms (or molecules) of a substance are present in a 1 micron-sized box at concentrations of 1 nM (nanomolar)? Note: 1 molar concentration corresponds to 1 mole of a substance confined in a 1-liter volume.

Note:
R= 8.31 J/mol·K
$N_A = 6.02 \times 10^{23}$ mol^{-1}
1 Liter = 10^3 cm^3

6. Starting from the differentials of the thermodynamic potentials:

$$dF = -SdT - PdV$$

$$dG = -SdT + VdP$$

$$dH = TdS + VdP$$

derive the Maxwell's relations:

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

Here, $F(V, T)$, $G(T, P)$, and $H(S, P)$ are the Helmholtz free energy, Gibbs free energy, and Enthalpy, respectively. You should use the definition of the total differential of a function, and that the mixed partial derivative of a function does not depend on the order of differentiation.

Modern Physics & Quantum Mechanics

7. a) The uncertainty in the position of an electron is given as 50 pm (picometers). What is the least uncertainty in any simultaneous measurement of the momentum of this electron?

b) Calculate the deBroglie wavelength of a 1.00 keV electron.

c) A photon has a wavelength of 0.20 nm (nanometers). Calculate its energy.

Note:
$h = 6.63 \times 10^{-34}$ J·sec
$m_e = 9.11 \times 10^{-31}$ kg
1 eV $= 1.6022 \times 10^{-19}$ J

8. The ground state of the simple harmonic oscillator (SHO) is geiven by: $\psi_0 = (\frac{m\omega}{\pi\hbar})^{1/4}e^{(-\frac{m\omega}{2\hbar}x^2)}$ Define a pair of operators: $a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$. Here, p is the momentum operator and x is the position operator. It can be shown that the first excited state of the oscillator is given by: $\psi_1 = A_1 a_+ \psi_0$ where A_1 is a normalizing constant.

a) Using the expression for the ground state of the SHO, find the first excited state.

b) Find A_1.