

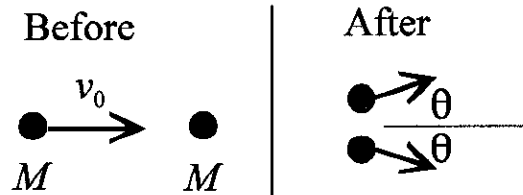
You may use mathematical tables and non-programmable calculators provided by the exam proctor.

### Classical Mechanics

1. A simple pendulum consists of a point mass  $M$  suspended from a massless string of length  $L$ .
  - a) Write the Lagrangian function for this system in terms of  $\theta$ , the angle which the string makes with the vertical. (Let  $\theta = 0$  when the mass is at its lowest point.)
  - b) Use Lagrange's equations to find the equation of motion (a differential equation for  $\theta$ .)
  - c) Using small-angle approximation (valid for  $\theta \ll 1$  radian), find the frequency of oscillation of the pendulum.

(If you have no experience with Lagrange's equations, find the frequency of small oscillations of the pendulum by another method.)

2. A particle of mass  $M$  moving at initial velocity  $v_0$  collides with a second particle of the same mass  $M$  which is initially at rest. After the collision, it is observed that both particles are moving at *equal angles*  $\theta$  relative to the original direction of motion, as shown in the figure. Do not assume the collision is elastic.



- a) Let the speeds of the two particles after collision be  $v_1$  and  $v_2$ . Using an appropriate conservation law, prove that  $v_1 = v_2$ .
- b) Find the speed of the particles after the collision.
- c) Find the total kinetic energy of the system after the collision, and compare it to the total kinetic energy before the collision.
- d) What would the angle  $\theta$  be if the collision is *elastic*?

## Electricity and Magnetism

3. A solid sphere of radius  $R$  has a non-uniform charge density  $\rho(r) = Cr^2$  (where  $C$  is a constant.)
- Find the electric field  $E(r)$  at points inside the sphere ( $r < R$ ).
  - Find the electric field  $E(r)$  at points outside the sphere ( $r > R$ ).
  - Find the potential  $V(r)$  at points outside the sphere ( $r > R$ ).
  - Find the potential  $V(r)$  at points inside the sphere ( $r < R$ ).
  - Sketch (qualitatively) a graph of  $V(r)$  versus  $r$ .
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4. a) Write the four Maxwell equations in differential form, in the presence of charge density  $\rho$  and current density  $\vec{J}$ , in free space ( $\epsilon = \epsilon_0, \mu = \mu_0$ ). Also write each equation in integral form, and give its usual name (if any.)
- b) Now assume that the charge density and current density in a region are both 0. Starting with Maxwell's equations in differential form, and looking up vector calculus identities as needed, show that the electric field in free space obeys a **wave equation**, and show explicitly from your derivation how the speed of the wave depends on the constants  $\epsilon_0$  and  $\mu_0$ . (Hint: Begin by taking the curl of both sides of suitable Maxwell equations.)

## Thermodynamics

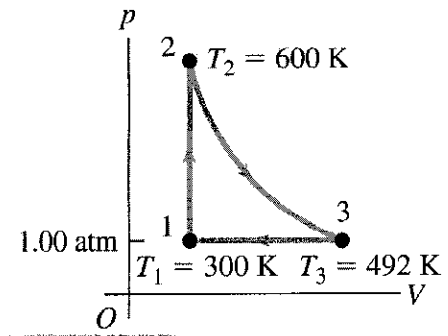
5. For carbon dioxide ( $\text{CO}_2$ ) gas, the constants in the van der Waals equation

$$\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

are  $a = 0.364 \text{ J}\cdot\text{m}^3/\text{mol}^2$  and  $b = 4.27 \times 10^{-5} \text{ m}^3/\text{mol}$ . The ideal gas constant is given by  $R = 8.31 \text{ J/mol}\cdot\text{K} = 0.08206 \text{ L}\cdot\text{atm/mol}\cdot\text{K}$ , and  $1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$ .

- If 1.00 mol of  $\text{CO}_2$  gas at 350 K is confined to a volume of  $400 \text{ cm}^3$ , find the pressure of the gas (in reasonable units) using (1) the ideal gas equation and (2) the van der Waals equation.
- Which equation gives a lower pressure? What is the percentage difference of the van der Waals result from the ideal gas result?
- Explain qualitatively the physical meaning of the constants  $a$  and  $b$ .
- If the volume expands to  $4000 \text{ cm}^3$ , do you expect the percentage difference to increase or decrease? Explain briefly.

6. A heat engine takes 0.350 mol of a diatomic ideal gas around the cycle shown in the  $pV$ -diagram. Process 1-2 is at constant volume, process 2-3 is adiabatic, and process 3-1 is at a constant pressure of 1.0 atm. ( $1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$ )  
For a diatomic ideal gas,  $C_V = 5/2 R$  and  $\gamma = C_P/C_V = 1.40$ .



- Find the pressure and volume at points 1, 2, and 3.
- Calculate the heat transfer  $Q$ , the work  $W$ , and the change in internal energy  $\Delta U$  for each of the three processes.
- Find the net work done by the gas in the cycle.
- Find the net heat flow into the engine in one cycle.
- Find the thermal efficiency of the engine.

## Quantum Mechanics/Modern Physics

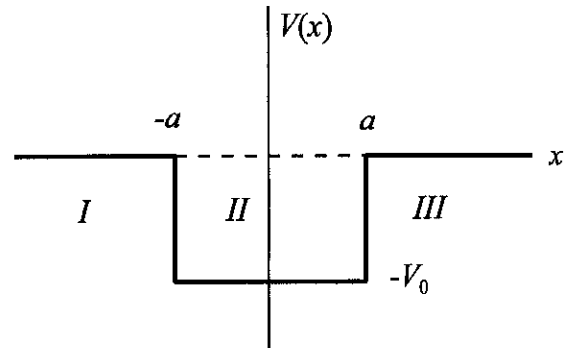
7. Ultraviolet light from a given source shines on a platinum cathode. It is observed that electrons are emitted from the cathode by means of the photoelectric effect. According to the Handbook of Chemistry and Physics, platinum has a work function of 5.65 eV.

Useful constants:  $hc = 1240 \text{ eV}\cdot\text{nm}$ ,  $e = 1.602 \times 10^{-19} \text{ C}$ .

- From the information given, determine the minimum or maximum wavelength of the UV light.
- Now a reverse potential difference is applied to the cathode. It is found that a retarding potential of 3.0 volts is sufficient to stop the photoelectric emission of electrons from the cathode. Find the wavelength of the UV light.
- If a steady current of 1.0 microampere ( $10^{-6} \text{ A}$ ) is measured flowing from the photocathode, how many electrons per second are being emitted? Does the answer depend on the wavelength of the light?

8. A particle of mass  $m$  is bound in a finite one-dimensional square well described by the potential energy function

$$\begin{aligned} V(x) &= 0 & x < -a & \text{(Region I)} \\ &= -V_0 & -a \leq x \leq a & \text{(Region II)} \quad (V_0 > 0). \\ &= 0 & x > a & \text{(Region III)} \end{aligned}$$



The energy of the particle has some definite negative value  $E_1$ , where  $-V_0 < E_1 < 0$ .

- In each of the 3 regions, write
  - the time-independent Schrödinger equation, and
  - the general form of the normalizable spatial wave function  $\psi(x)$  of the particle, assuming that  $\psi(x)$  is an even function of  $x$  (i.e.  $\psi(-x) = \psi(x)$ ). Calculate the values of all the constants in your function that are determined by the given quantities.
- Make a qualitative sketch of  $\psi(x)$  versus  $x$ .
- Apply the boundary conditions at  $x = -a$  and  $x = a$  to write a set of equations which can (in principle) be solved to find the unknown constants in the wave function. (Do not solve.)
- Write the time-dependent wave function  $\Psi(x,t)$  in terms of  $\psi(x)$ . (Do not substitute for  $\psi(x)$ .)