Synthetic correlation-based modified signed-digit trinary logic processing

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Abstract. An optical implementation using a correlation technique for modified signed-digit (MSD) trinary logic processing is presented. In particular, a synthetic matched filter (SMF) correlator model is used to demonstrate the realization of the carry-propagation-free trinary MSD addition. It is shown that proper encoding of the MSD numerals are of utmost importance for the correlator model to work. The developed method is expected to have far-reaching application in the optical higher radix numeric and logic processing. © *1999 Society of Photo-Optical Instrumentation Engineers.* [S0091-3286(99)01003-X]

Subject terms: modified signed-digit; synthetic matched filter; correlation; trinary logic.

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1 Introduction

Propagation of carry bit is an inherent problem in an arithmetic addition process that slows down the speed of operation of digital processing for its sequential nature. Techniques such as carry-look-ahead and carry-save attempt to overcome this problem[1] at the cost of increased fan-in and the resultant higher level of gate complexity.

Carry-propagation-free addition using modified signed-digit (MSD) numbers alleviate[2] such problems and is therefore readily amenable to parallel operation. Optical computation with its parallel processing capability is therefore a viable solution for these applications.[3] Higher radix representations, such as trinary based arithmetic, is even more attractive because of its higher storage density and reduced logic requirement.[4,5] In addition, computing systems using MSD logic are scalable to higher sizes by simple replication of the hardware.

Many different optical realizations such as optical content-addressable memory,[6,7] optical shadow-casting,[8] symbolic substitution,[9–12] and holograms[13] have been attempted for MSD-based logic processing. This paper takes a different route, and that is the use of correlation in MSD processing. Optical correlation is a widely used technique in the pattern recognition paradigm. Correlation provides the degree of matching between two signals.[14] This technique, as it applies to arithmetic-logic unit, finds the matching between input and stored reference patterns, which is then thresholded to give desired binary outputs.[15] The matched filter[14] (MF) and the joint Fourier transform[16] (JFT) are the two established setups for correlation. In this paper, we propose to develop a synthetic matched filter (SMF) from the superposition of the phase-only information of some training patterns. This SMF is used as the reference filter. The codes of the individual minterms of the logic function are used as the inputs. The coding scheme is found to be vital in obtaining the desired outputs.

2 MSD Trinary Number

A signed-digit trinary number X can be represented by

$$X=\sum_{i=0}^{n} x_i R^i, \qquad (1)$$

where the radix $R=3$ and the numeral x_i can take any value from the set $\{\bar{2},\bar{1},0,1,2\}$. Multiple representation of the same number leads to redundancy in the MSD system. For example, number of combinations to make a two-digit MSD number using these five numerals is 25 (5^2), whereas 17 $(2*3^2-1)$ of them are unique. Therefore, the redundancy here is 32% (8 out of 25). It is this inherent redundancy in MSD number representation that is exploited to derive rules for carry-propagation-free addition.

When two single-digit trinary MSD numbers are added, a carry is generated whenever the result is greater than or equal to the radix. Thus the combinations (1,2), (2,2), $(\bar{2},\bar{1})$ or $(\bar{2},\bar{2})$ results in a carry. The generation of carry can be avoided by mapping the two digits in question into an intermediate sum and an intermediate carry such that the n'th intermediate sum and the $(n-1)$'th intermediate carry never form any one of the aforementioned four combinations. Table 1 shows the truth table of intermediate carry and sum output generated from the addition of two single-digit trinary MSD numbers.[17] Note that there are a total of 25 minterms (enumerated from 0 to 24). Similar values for

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Table 1 Truth table for trinary MSD addition.

Type	Minterm Number	Addend (A_i)	Augend (B_i)	Intermediate Carry	Intermediate Sum
1	0	0	0		
	1	1	$\bar{1}$		
	2	$\bar{1}$	1	0	0
	3	2	$\bar{2}$		
	4	$\bar{2}$	2		
2	5	0	1		
	6	1	0		
	7	2	$\bar{1}$	0	1
	8	$\bar{1}$	2		
3	9	0	$\bar{1}$		
	10	$\bar{1}$	0		
	11	1	$\bar{2}$	0	$\bar{1}$
	12	$\bar{2}$	1		
4	13	0	2		
	14	2	0	1	$\bar{1}$
	15	1	1		
5	16	0	$\bar{2}$		
	17	$\bar{2}$	0	$\bar{1}$	1
	18	$\bar{1}$	$\bar{1}$		
6	19	1	2	1	0
	20	2	1		
7	21	$\bar{1}$	$\bar{2}$	$\bar{1}$	0
	22	$\bar{2}$	$\bar{1}$		
8	23	2	2	1	1
9	24	$\bar{2}$	$\bar{2}$	$\bar{1}$	$\bar{1}$

carry and sum outputs are grouped together, resulting in nine groups of these minterms. This symmetry in groups of the truth table is exploited here for a possible optical implementation.

3 Synthetic Correlation-Based MSD Adder Realization

An SMF correlator is now developed that is used for the MSD adder realization.

The cross-correlation of a reference function $h(x)$ with an input function $f(x)$ is given by

$$g(x)=f(x)\otimes h(x)=\int_{-\infty}^{\infty} f(y)h^*(y-x)\mathrm{d}y. \tag{2}$$

This correlation operation, in Fourier domain, is computed as

$$g(x)=\mathcal{F}^{-1}[F(\xi)H^*(\xi)]. \tag{3}$$

Here $F(\xi)$ is the Fourier transform of the input function; $H^*(\xi)$ is the complex conjugate of the Fourier transform of the reference function; $\mathcal{F}^{-1}$ stands for the inverse Fourier transform; and $H^*(\xi)$ is the matched filter, which is used as a stored filter here.

A matched filter by definition results in the highest signal-to-noise ratio (SNR), when SNR is defined by the ratio of the average correlation peak to the variance of the correlation peak. The seminal work by Oppenheim and Lim[18] proved that phase information is more important than the amplitude information in the case of MF based recognition. This result led to the introduction of phase-only filter (POF) based matched correlation.[19] When compared to the classical matched filter, a POF results in a sharper correlation peak[19] and results in 100% light efficiency. In this paper, we use the POF technique to come up with the SMF.

The steps involved in the MSD adder realization are:

- **Step 1** Identification of outputs from Table 1.
- **Step 2** Identification of minterms for each output.
- **Step 3** Encoding of MSD numerals.
- **Step 4** Encoding of minterms consisting of MSD numerals.
- **Step 5** Constructing an SMF for each output.
- **Step 6** Interpreting correlation result.

Each step is explained in the following.

3.1 *Step 1*

The trinary values from intermediate carry and sum outputs are 0, 1, and $\bar{1}$, as seen from Table 1. We are concerned with only the nonzero outputs in the proposed SMF. These are identified as four variables as follows: (1) carry resulting a $+1$ (C_p), (2) carry resulting a -1 (C_n), (3) sum resulting a $+1$ (S_p), and (4) sum resulting a -1 (S_n).

3.2 *Step 2*

Each input MSD pair is called a minterm. For example, minterm number 1 (as enumerated in Table 1) consists of the MSD pair $\{1,\bar{1}\}$. Now using sum-of-products representation of minterms,[1] the four output variables can be expressed as,

$$\begin{aligned} C_p&=\sum 13,14,15,19,20,23,\\ C_n&=\sum 16,17,18,21,22,24,\\ S_p&=\sum 5,6,7,8,16,17,18,23,\\ S_n&=\sum 9,10,11,12,13,14,15,24. \end{aligned} \tag{4}$$

3.3 *Step 3*

Each of the five trinary MSD numerals is encoded as a 1×5 vector as follows:

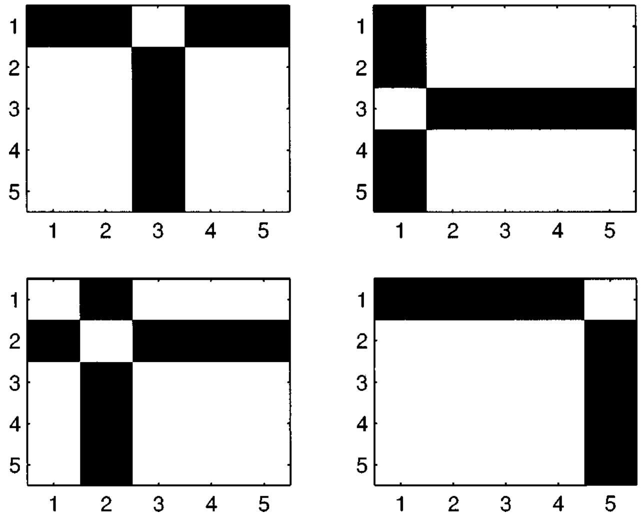

Fig. 1 Codes for the minterms 13, 14, 15, and 16.

$$0\rightarrow[HLLLL],$$
$$1\rightarrow[LHLLL],$$
$$2\rightarrow[LLHLL], \tag{5}$$
$$\bar{1}\rightarrow[LLLHL],$$
$$\bar{2}\rightarrow[LLLLH].$$

Here the optical meaning of H and L pixels are on and off, respectively.

3.4 *Step 4*

As mentioned earlier, a minterm involves two such MSD numerals. The code for each minterm is obtained by taking the outer product of the corresponding numerals, resulting in a 5×5 code for each minterm. Some nonlinear operation after the outer product may actually be necessary, as explained in the next section. Figure 1 shows the codes for four such minterms (numbers 13, 14, 15, and 16). The outer product formulation resulted in a very interesting pattern: two minterms having the same numerals but in different orders result in codes that are transpose of each other. For example, minterm 13 and 14, as shown in the first row of Fig. 1, are transposes of each other. Incidentally, in the proposed SMF, because of this transpose characteristics, both the codes give same correlation peak values, which is very desirable in setting a threshold in the correlation plane. Note that the codes in Fig. 1 are obtained by setting $H=+1$ and $L=-1$ in the outer product formulation.

3.5 *Step 5*

An SMF is constructed next for each of the output variables using the synthesis of aforementioned minterm codes. Suppose, an output variable has n coded minterms that result in nonzero values. Let F_i be the Fourier transform of minterm i, where $F_i=|F_i|e^{j\phi_i}$. The SMF corresponding to this output is then synthesized by,

$$H^*=\left[\sum_i^n \frac{F_i}{|F_i|}\right]^*. \tag{6}$$

Here * represents the complex conjugate operation. For example, to construct the SMF corresponding to C_p, we take the codes of the minterms 13, 14, 15, 19, 20, and 23 as developed in step 4, which are then processed according to Eq. (6).

3.6 *Step 6*

Correlation of an SMF with the code of an input minterm is carried on next, according to Eq. (2). An input that gives a zero value for an output variable should result in smaller correlation compared to an input that gives a nonzero value. At this stage, a threshold formulation is necessary to distinguish between the zero and nonzero values.

4 Simulation Results

The minterm codes are zero-padded to result in an SMF of size 32×32. The effect of coding scheme is extremely critical in this application. We report the results of different coding schemes and synthesis process. In the simulation, we considered two different values of L and H, as used in the coding of MSD numerals. One pair is $L=-1$, $H=+1$, while another pair is $L=\exp(j\pi/2)$, $H=e^{j0}$. We also considered two different realizations of actual coded minterms after the outer product formulation: (1) without any nonlinear postprocessing and (2) with nonlinear postprocessing that resulted in a pixel value 1 or 0 in the final code. Finally, the ''overloading'' problem of synthesis process is addressed and a minimization formula in the minterm synthesis is also proposed. This is somewhat related to the overloading of an associative memory that dictates that for efficient retrieval, an associative memory is constrained to have a maximum capacity; any pattern added beyond that capacity actually reduces the retrieval rate.

Let us now compare the results of three different coding and synthesis schemes. In all of these, we used $L=\exp(j\pi/2)$, $H=e^{j0}$.

4.1 *Scheme 1*

Without nonlinear postprocessing of minterm codes and without any minterm minimization, in other words, the outer product results of step 4 are used directly for the codes without any postprocessing. Figure 2 shows the correlation results of all 25 minterms with the four SMFs constructed for the outputs C_p, C_n, S_p, and S_n as depicted in Figs. 2(a), 2(b), 2(c), and 2(d), respectively. The peaks with bubbles represent the desired correlation value corresponding to the minterms resulting in nonzero value for an output variable. In other words, the desired correlation peaks correspond to the minterms included in the output variables, as mentioned in Eq. (4). For example, in Fig. 2(a) peaks with bubbles correspond to the minterm numbers 13, 14, 15, 19, 20, 23. Note that although, in general, these minterms have higher correlation peak values, it is hard to set a threshold to distinguish them from the minterms generating a zero result. From Eq. (4), we find that there is a total of 28 active minterms resulting in 28 desired correlation values. With a threshold value of 80 in Fig. 2(a), 5 out of 6 desired peaks

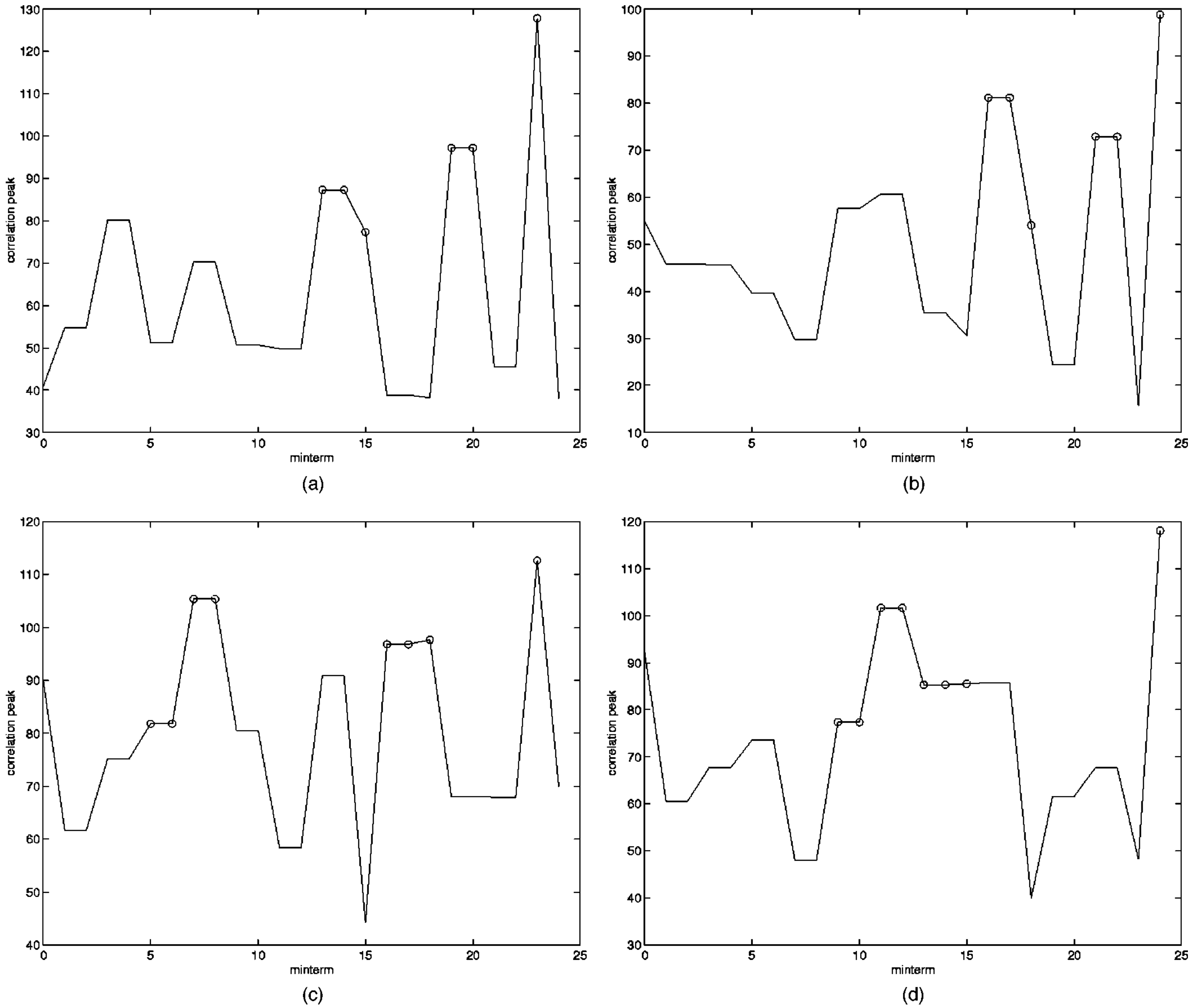

Fig. 2 Correlation peak output from the SMFs: (a) C_p, (b) C_n, (c) S_p, and (d) S_n. No nonlinear postprocessing of minterm codes. No minimization of minterms.

can be detected. Similarly, a threshold of 70 detects 5 out of 6 desired peaks in Fig. 2(b), whereas, 6 out of 8 can be detected from Fig. 2(c) (using threshold of 90), and finally 3 out of 8 can be detected from Fig. 2(d) (using a threshold of 80). With these thresholds, therefore, a total of only 19 out of these 28 peaks can be distinguished. This problem is alleviated in the following schemes.

4.2 *Scheme 2*

With nonlinear postprocessing of minterm codes and without any minterm minimization, here, the outer product codes go through a nonlinear processing that results in a 1 or 0 for each pixel code. The particular nonlinearity used here is a simple binarization of the outer product values. These coded minterms then use the same synthesis process to construct the SMFs. Correlation results are shown in Fig. 3 in which 26 out of 28 can be distinguished now. A threshold value of 90 can be used to detect all six desired peaks for C_p, as shown in Fig. 3(a). Similarly a threshold of 125 is sufficient to detect all desired peaks for S_p and S_n, as shown in Figs. 3(c) and 3(d). Two desired minterms for the C_n output gave correlation peaks smaller than the undesired ones, as shown in Fig. 3(b).

4.3 *Scheme 3*

With nonlinear postprocessing of minterm codes and with the minterm minimization, further improvement is obtained by efficiently minimizing the number of training patterns in the formulation of SMFs. The fact that the final correlation value includes the superposition of the correlation values of training codes suggests that minimizing the number of training codes will lead to less cross-correlation, thus better discrimination. To realize scheme 3, we exploit the symmetry of the MSD truth table for the minimization. Note that some of the training pairs in the original sequence are transposes of each other, for example, patterns 13 and 14, as

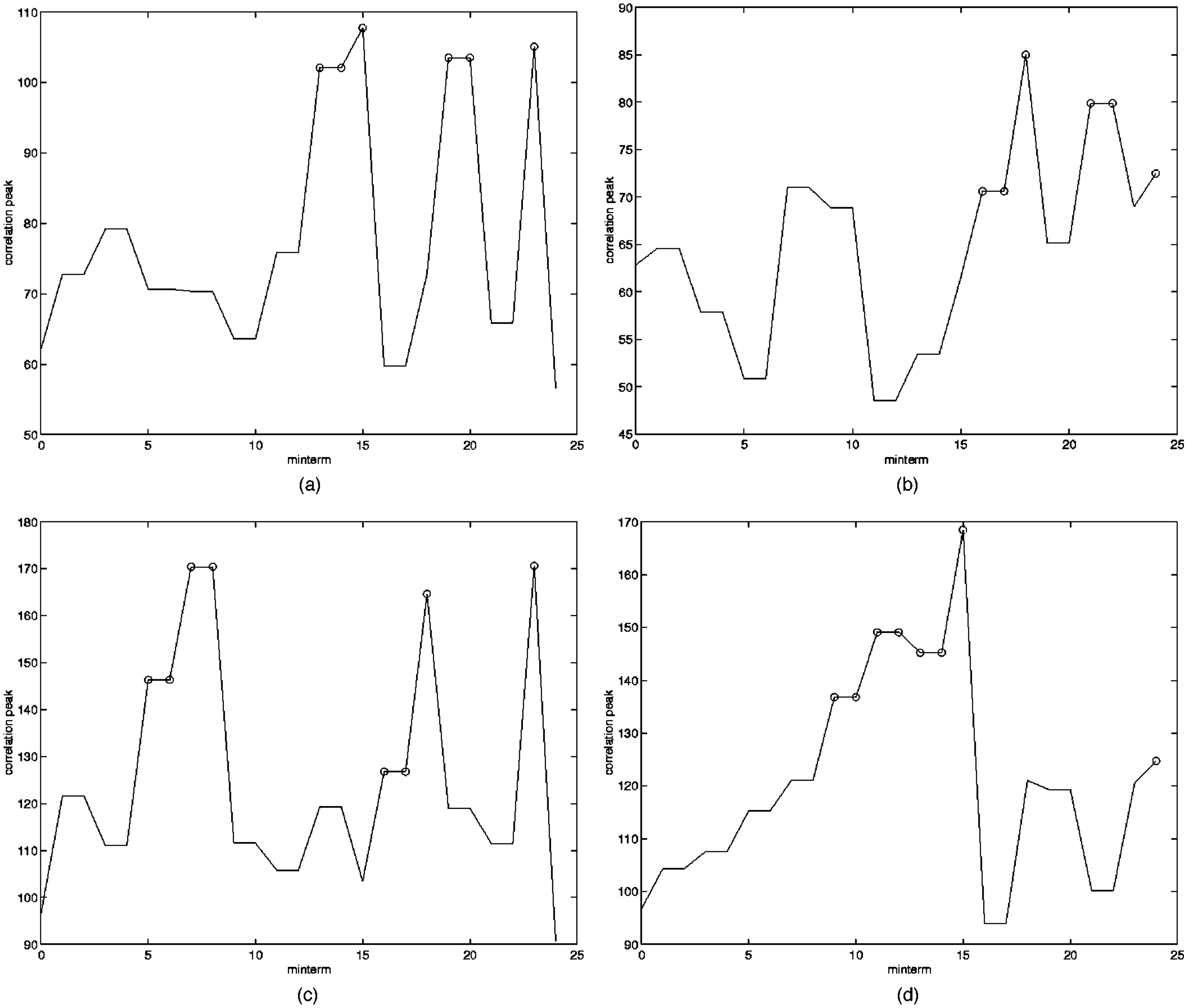

Fig. 3 Correlation peak output from the SMFs: (a) C_p, (b) C_n, (c) S_p, and (d) S_n. With nonlinear postprocessing of minterm codes. No minimization of minterms.

shown in Fig. 1. With this formulation the minterm synthesis of the output SMFs are changed as follows:

$$\begin{aligned} C_p &= \sum 13,15,19,23, \\ C_n &= \sum 16,18,21,24, \\ S_p &= \sum 5,7,16,18,23, \\ S_n &= \sum 9,11,13,15,24. \end{aligned} \tag{7}$$

The results of this reformulation is shown in Fig. 4. Note that the correlation peak values for the desired minterms are well-separated now from the undesired ones. In order to quantify the results, we define two metrices. Worst case discrimination ratio (WCDR) is defined as follows:

$$\mathrm{WCDR} = \frac{\text{minimum desired correlation peak}}{\text{maximum undesired correlation peak}}. \tag{8}$$

Table 2 enumerates WCDR values of the output variables for different coding and synthesis schemes. Another parameter named average signal-to-noise ratio (SNR) for this application is defined as follows:

$$\mathrm{SNR} = 10 \log \frac{\text{average correlation energy for desired peaks}}{\text{average correlation energy for undesired peaks}}. \tag{9}$$

Table 3 enumerates the SNR values.

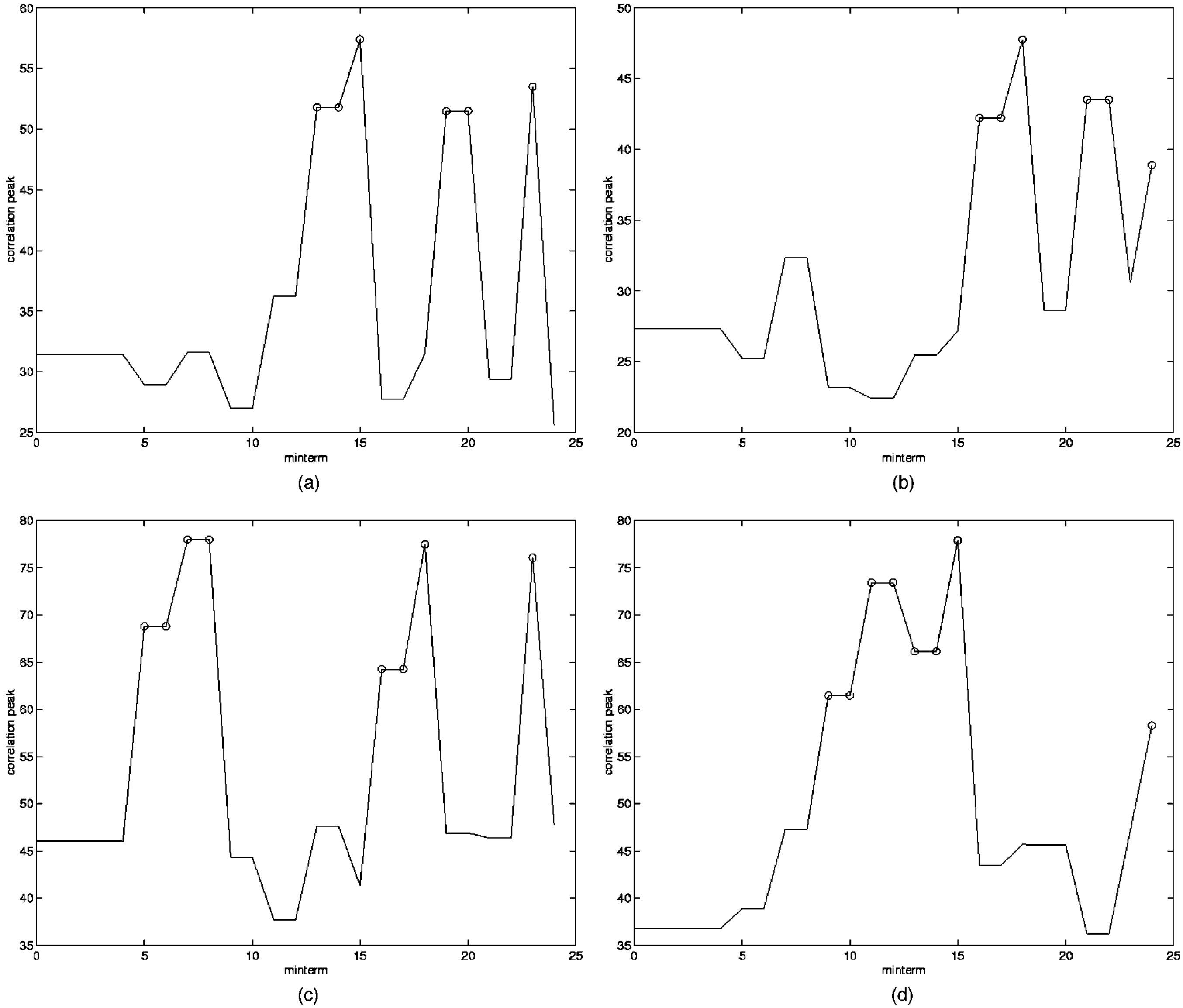

Fig. 4 Correlation peak output from the SMFs: (a) C_p, (b) C_n, (c) S_p, and (d) S_n. With nonlinear postprocessing of minterm codes and with minimization of minterms.

5 Conclusion

A proposal for correlation-based optical realization for trinary MSD addition is presented. Effects of different coding schemes and minterm synthesis were demonstrated. Worst case discrimination ratio and SNR metrics were defined to quantify the better performance of the proposed synthetic correlator in distinguishing desired minterms from the undesired ones and thus implementing the MSD addition truth table. The methodology discussed here can be used to optically realize any generalized table look-up problem.

Table 2 Worst-case discrimination ratio from correlation results.

Coding and Synthesis Scheme	Output Variables			
	C_p	C_n	S_p	S_n
Scheme 1	0.97	0.89	0.90	0.84
Scheme 2	1.3	0.99	1.05	1.03
Scheme 3	1.42	1.22	1.35	1.24

[a]Scheme 1: No nonlinear postprocessing of minterm codes; No minimization of minterms.

[b]Scheme 2: With nonlinear postprocessing of minterm codes; No minimization of minterms.

[c]Scheme 3: With nonlinear postprocessing of minterm codes and with minimization of minterms.

Table 3 Average SNR from correlation results.

Coding and Synthesis Scheme	Output Variables			
	C_p	C_n	S_p	S_n
Scheme 1	5.07	5.17	2.67	2.8
Scheme 2	3.55	1.95	2.8	2.4
Scheme 3	4.81	4.08	4.08	4.26

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Abdul Ahad S. Awwal: Biography and photograph appear with the special section guest editorial in this issue.

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