IMAGE SMOOTHING WITH MINIMAL DISTORTION

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ABSTRACT

An innovative method that enhances detail in digital images by smoothing image pixels while introducing minimal distortion is described and tested. In particular, a 14 by 14 pixel region of a digital image is smoothed using a constrained Gaussian radial basis function method. This method centers on each pixel a Gaussian distribution of amplitude such that the sum of all distributions correctly reproduces the gray level of each pixel. To assess the method the distortion of the smoothed image as measured by the deviation of its power spectrum from that of the unsmoothed image is determined as a function of the Gaussian distribution width, and comparisons are made with bilinear interpolation, a conventional convolution smoothing technique. The new method is capable of removing more "pixel noise" while introducing less image distortion, thus permitting the detection and examination of otherwise hidden detail in digital images. Example include the detection and assessment of enemy weapons in military images and cancerous tumors medical images.

1. INTRODUCTION

Numerous digital image smoothing techniques are available that remove blockiness due to image quantization or pixelation [e.g., Pratt, 1991]. These techniques usually involve smoothing a square-pixel image by convolving it with a kernel function. Image reconstruction techniques may also be used to approximate an image in which some portions are missing or obscured [Gilbert and Gustafson, 1993]. Typical approaches to the reconstruction problem include the use of interpolation methods such as cubic B splines. However, with these methods the pixels typically must be on a uniform grid, and the reconstructed pixels are often not consistent with pixels reconstructed by known image formation processes. Radial basis function interpolation and approximation techniques can avoid these limitations [Gustafson et al., 1992, 1994].

Variable degrees of image smoothness can be achieved using radial basis function interpolation techniques. Smoothness can be specified by an integrated squared second derivative measure. For a given smoothness, radial basis function interpolation using circular Gaussian basis functions yields a spatial frequency power spectrum that tends to minimize the integrated squared deviation from the power spectrum of the data before interpolation [Poggio and Girosi, 1990]. This kind of interpolation can smooth to any specified degree while retaining more image detail than conventional convolution smoothing methods.

0-8194-1841-2/95/$6.00

2. RADIAL BASIS FUNCTION SMOOTHING

An image may be smoothed using Gaussian radial basis function (RBF) interpolation with a single variable standard deviation. The method involves centering a Gaussian function on each pixel of the image so that

$$z(x,y) = A(i,j)\exp\left[-\frac{\left((x-i)^2+(y-j)^2\right)}{2\sigma^2}\right] ,$$

where i and j represent pixel position, A(i, j) is the amplitude of the Gaussian function on pixel (i, j), and σ is standard deviation. The Gaussian functions are summed to form

$$g(x,y) = \sum_{j=1}^{n}\sum_{i=1}^{n} A(i,j)\exp\left[-\frac{\left((x-i)^2+(y-j)^2\right)}{2\sigma^2}\right]$$

where n^2 is the number of pixels. The gray value G(i, j) of pixel (i, j) is required to equal g(x, y) integrated over this pixel so that

$$G(i,j) = \int_{j-0.5}^{j+0.5}\int_{i-0.5}^{i+0.5} g(x,y)\,dx\,dy \text{ , for i, j = 1, 2, ..., n.}$$

With σ specified a system of linear equations is found with the weighting factors A(i, j) as the only unknowns. This system is solved for the A(i, j), which are substituted in z(x, y).

3. SMOOTHED IMAGE COMPARISONS

Figure 1a shows a 40 by 40 pixel image segment of a human eye. The original image segment of 20 by 20 pixels is replicated (i.e., zero-order hold is performed) with a magnification of 2 to obtain this result. Figures 1b and 1c show, respectively, the original image smoothed from 20 by 20 pixels to 40 by 40 pixels using bilinear [e.g., Pratt 1991] and Gaussian RBF interpolation. Histograms of the power spectra are displayed for comparison. Note that for replication there are high low frequency components due to smooth gray values from zero-order hold: the subpixels of a particular pixel of the zero-order hold image have the same gray values. Bilinear interpolation reduces this smoothness to yield more detailed information from the image. RBF interpolation performs well in terms of keeping both the smooth and the detailed information. In particular, note that the power spectrum has a larger number of pixels with frequencies smaller than 0.1 and larger than 0.6 when compared with bilinear interpolation.

4. ASSESSMENT OF SMOOTHED IMAGE DISTORTIONS

RBF interpolation is compared with bilinear interpolation with regard to the minimization of distortions. The test image is a 14 by 14 pixel segment of a FLIR (forward looking infrared) image designated z_0. Using bilinear interpolation z_0 is expanded to a 28 by 28 image z_l. The power spectrum of these images is

$$\mathbf{I}_o(\xi,\eta) = |F(z_o)|^2 \quad \text{and}$$
$$\mathbf{I}_l(\xi,\eta) = |F(z_l)|^2,$$

where F indicates the discrete Fourier transform. The distortion between the original and the bilinear interpolated image is measured by

$$d_{ol} = \sqrt{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[\mathbf{I}_l - \mathbf{I}_o]^2 \, d\xi d\eta} \quad .$$

Using RBF interpolation as discussed above to expand z_o to a 28 by 28 image z_r requires the solution of $28^2 = 784$ linear equations in the same number of unknowns, which are the amplitudes of each Gaussian function on each pixel. The power spectrum is

$$\mathbf{I}_r(\xi,\eta) = |F(z_r)|^2$$

and the distortion is measured by

$$d_{or} = \sqrt{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}[\mathbf{I}_r - \mathbf{I}_o]^2 \, d\xi d\eta} \quad .$$

This procedure is repeated for different values of σ, and the two distortions d_{ol} and d_{or} are plotted versus σ.

Figure 2a shows the 28 by 28 expanded test image and Figure 2b shows the bilinear interpolation of this image. Figures 3a to 3c are interpolated images using RBF smoothing with different standard deviations. It is apparent that as the standard deviation increases the image becomes smoother.

The smoothed image with minimal distortion may be identified from Figure 4, which plots the distortion measures d_{ol} and d_{or} versus σ. It is apparent that for standard deviations between about 0.7 and 1.1 the distortion between the original image and the RBF-smoothed image is less than the distortion between the bilinear-smoothed image and the original image. The optimum standard deviation (using the d_{0r} measure) is about 0.9.

5. CONCLUSION

It is shown that digital images can be smoothed using a Gaussian radial basis function interpolation method that preserves gray levels while introducing minimal distortion, where distortion is measured by the integrated squared deviation of the power spectrum of the smoothed image from that of the unsmoothed image. RBF smoothing is shown to be capable of less distortion than standard bilinear interpolation, but it requires the solution of as many simultaneous linear equations in as many unknowns as there are pixels, so that in practice its application is limited to smoothing small image regions.

6. REFERENCES

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(a)

(b)

(c)

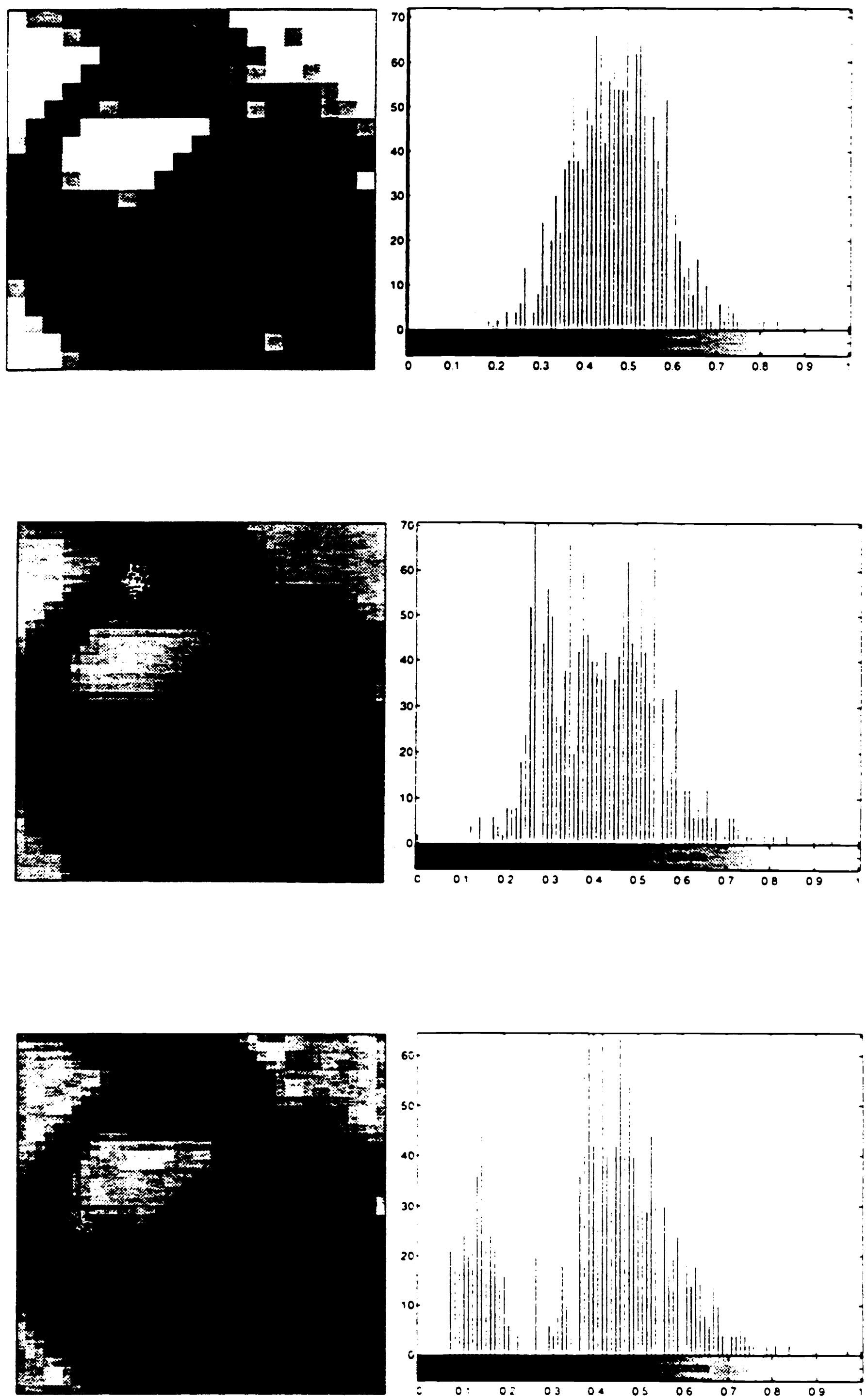
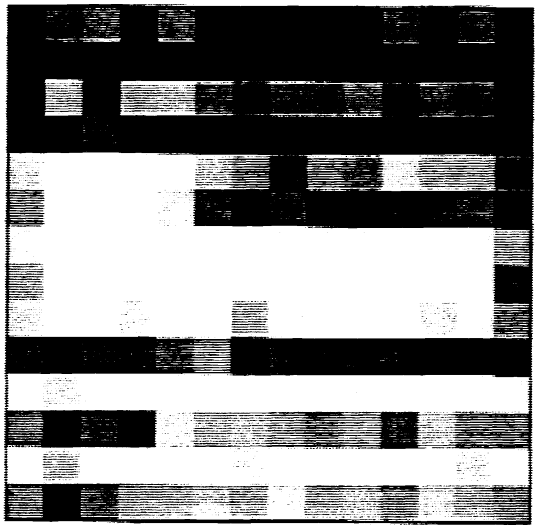


Figure 1. Interpolated eye image and its power spectra for (a) replication, (b) bilinear interpolation, and (c) RBF interpolation.

(a)



(b)

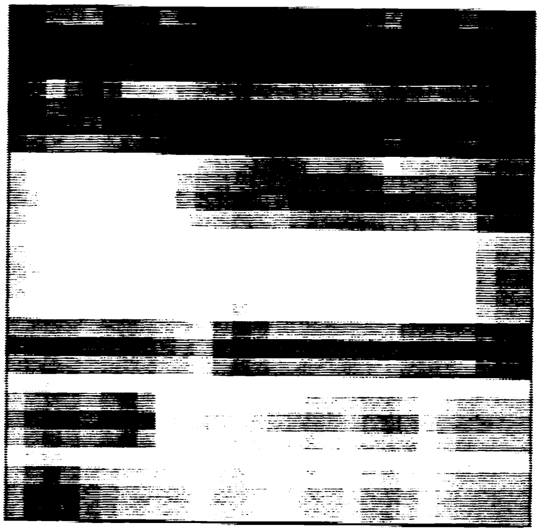


Figure 2. FLIR test image (a) replicated, and (b) bilinear interpolated.

(a)

(b)

(c)

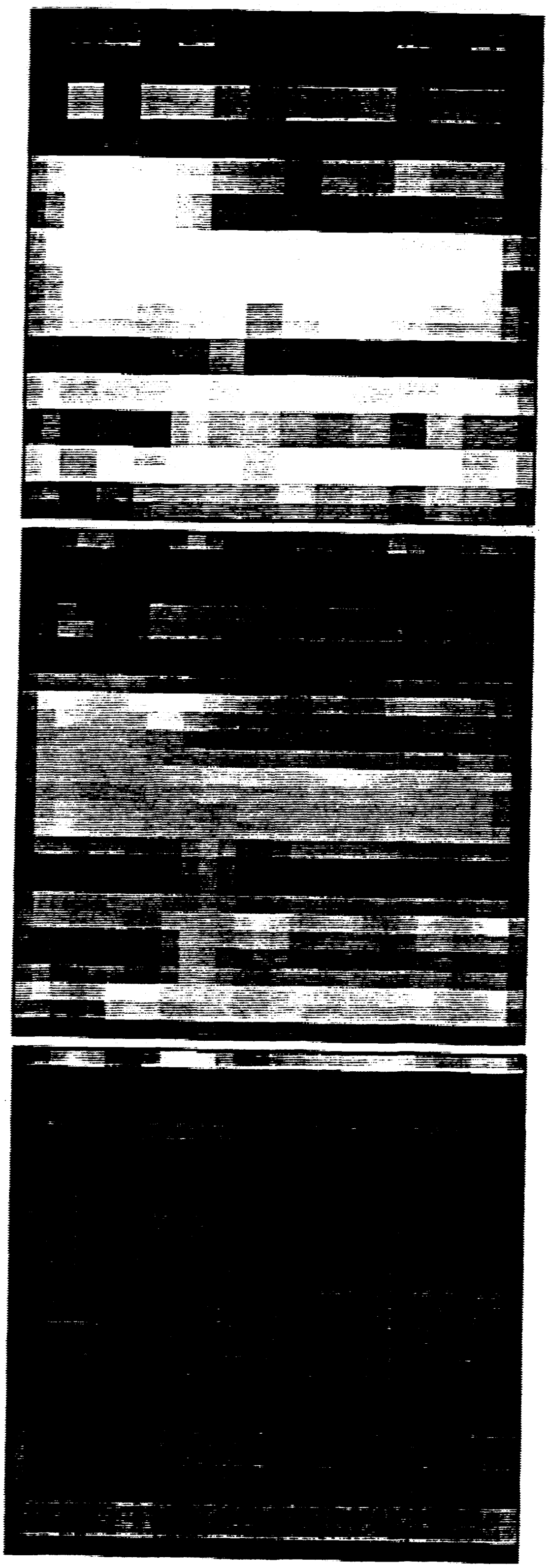


Figure 3. FLIR test image smoothed using Gaussian RBF interpolation with (a) $\sigma = 0.3$, (b) $\sigma = 0.9$, and (c) $\sigma = 1.5$.

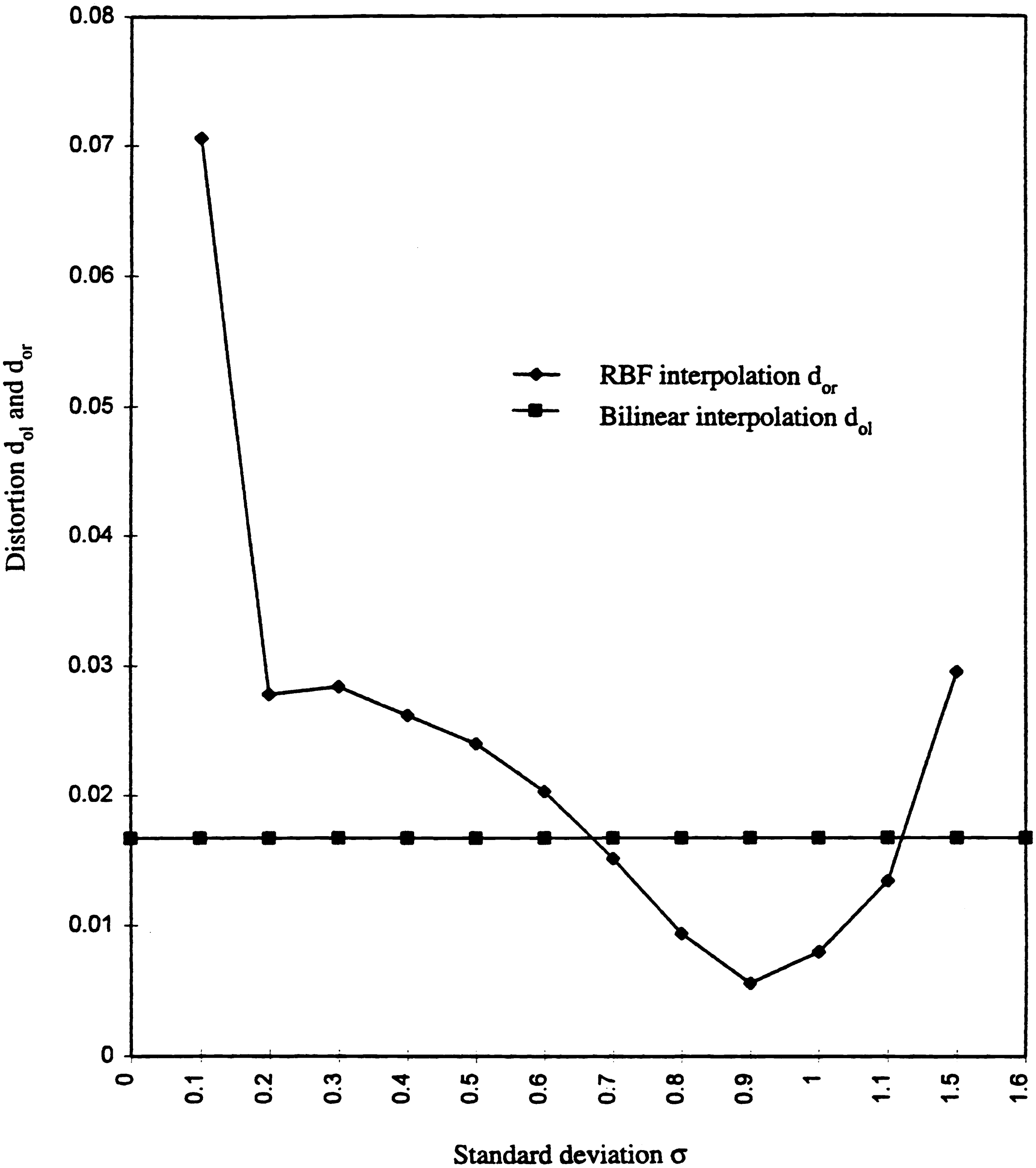


Figure 4. Comparison of distortion for RBF interpolation and bilinear interpolation for different standard deviations.