A COMPOSITE MAPPING TECHNIQUE FOR SIMPLIFICATION OF MULTI-VARIABLE BOOLEAN EXPRESSIONS

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ABSTRACT

A composite mapping technique using a newly proposed logic minimization scheme (KH-map) has been investigated here. This paper presents an extended feature of KH-map that can combine multiple maps for better representation of switching functions within limited space, for a relatively large number of variables. The combined KH-map can be efficiently used to simplify Boolean expressions to be realized in two-level logic. The technique is simpler, more generalized and more efficient than conventional minimization methods and is easily applicable for any number of variables.

I. INTRODUCTION

The problem of minimizing switching expressions has been studied extensively in the literature [1-10]. Among all of them K-map is most widely used. But it appears to be very cumbersome for a large number of variables. Quine-McCluskey method, on the other hand can systemetically solve logic expressions of large number of variables, but it also incurs huge computation. A new method called KH-map has recently been proposed [11,12], that presents a very intutive approach to the logic minimization problem.

In this paper concatenation of multiple KH-map will be introduced with its application in finding minimal sum of product. For each combination of n variables or bits there exists exactly n different binary codes with Hamming distance one, i.e., each binary combination has n adjacent combinations. Representation of such adjacency relation by Karnaugh map is very difficult for large values of n. The proposed KH-map method, however, will be able to overcome such difficulties.

II. KH-MAP

A KH-map is a modified form of a truth table in which the arrangement of the combinations is particularly convenient, that exploits circular symmetry. The new maps for functions of three and four variables are shown in Fig. 1 and Fig. 2 respectively. Proposed map for n variables containing 2^n cells equally spaced within a circle represents all possible combinations of n variables.

Line(s) passing through the center that have been used for dividing cells, are

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called MSB line(s). If v is the ith MSB variable then ith MSB lines or simply v-lines will be max(1, 2^{i-2}) in number. In Fig. 2 w'-w and x'-x lines are first and 2nd MSB-line respectively while y-y and y'-y' lines are 3rd MSB-lines.
Those v-lines divide all the cells equally as formed previously by other MSB variables of v. In Fig. 2 w'-w and x'-x MSB-lines divide the circle into four cells and two 3rd MSB-lines y-y and y'-y' divide the circle into 8 equal cells, i.e., 4 more new cells is formed. The combination contained in a cell can be calculated using following simple KH-algorithm:

SIMPLE KH-ALGORITHM

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1. Let C1, C2, . . ., C2n represent all cells.
2. set C1 = 0, m=1
       for i= 1 to n do
           for k = 1 to m do
                   Cm+k = Cm-k+1 + m
           end-for
                   m = 2*m
       end-for
```

An alternative approach is to add 2^k with each cell formed by the mirror image of the previous 2^k cells, where $0 <= k < n$ and initialize the first cell with 0. The process is quite similar to the process of gray code generation.

The center divides each v-line into two parts. Two parts of the 1st MSB (w'-w in fig. 2) line are marked by positive(w) and negative(w') parts containing positive and negative end respectively. All end points of v-lines are sequentially marked by v, v', v, ... v' in clock-wise direction starting from the negative end of 1st MSB line. In Fig. 2 end points of all 3rd MSB-lines are marked sequentially by y, y', y, y'. Parts containing negative ends (v' points) are negative v-lines while other are positive v-lines. If one moves anticlockwise from a cell then one will at first get positive v-line among all v-lines, provided variable v in the cell is 1, otherwise it will be the negative v-line. Calculating only the end points of the MSB lines the bit combination in a particular cell can be calculated. For example in Fig. 2 if one moves anticlock-wise from the cell containing 11 then he will find z, x', z', y, z, w, z', y', z, x and so on as the end points of the MSB-lines. If he collects end points that come first then he will get z, x', y, and w, which are the variable combination in that cell (11). All MSB lines can be drawn or only the end points can be marked. The ith MSB is also the (n-i)th LSB line.

TRUE, FALSE, DON'T CARE vertex: The mid points of an arc of a cell are marked by • (small circle), x, or - sign depending on function values TRUE, FALSE or DON'T CARE respectively for the combination in that cell. The points marked by • are identified as TRUE or 1 vertices, points marked by x are FALSE or 0 vertices andthe rest as DON'T care vertices.

Theorem 2.1 If L denotes the line joining vertices V and V' then V and V' will be adjacent by M_j MSB bit (i.e they differ by only M_j MSB bit) if and only if the followings are true.

a. If N is the total number M_k-MSB lines that have been intersected by line L then N must be even, where k<>j.
b. L intersects only one M_j-MSB line.
c. L never intersects M_l-MSB line where l<j or M_l is MSB of M_j.
d. M_j-MSB line is bisector of L.

Definition 2.2 Two vertices V1 and V2 are called adjacent about M MSB line if only M MSB variable differ between them and they can be detected satisfying conditions in theorem 2.1.

Definition 2.3 A polygon consists of a collection of 2^m vertices each adjacent to m vertices of the collection, is called a KH-polygon, and the KH-polygon is said to cover these vertices, where, 0 <= m <= n. Two consecutive vertices of the KH-polygon must be adjacent.

Theorem 2.4 Two KH-polygon P and P′ of 2^m vertices each can be combined to form a new KH-polygon P″ of 2^{m+1} vertices if and only if for any vertex V in P, P′ contains unique adjacent vertex of V that is no present in P and vice versa.

Theorem 2.5 Minterm for a KH-polygon of 2^m vertices will contain n-m literals.

III. SIMPLIFICATION AND MINIMIZATION OF FUNCTIONS

Each KH-polygon of 2^m vertices can be expressed by a product containing (n - m) literal, where n is the number of variable in the expression. Function f can be expressed as a sum of those product terms that correspond to the KH-polygon(s) necessary to cover all its 1 vertices. The number of product terms in the expression for f is equal to the number of KH-polygon. In order to obtain a minimal expression, all 1 vertices must be covered with the smallest possible number of KH-polygons, such that each KH-polygon is as large as possible. A KH-polygon contained in another larger KH-polygon must never be selected. If there is more than one way of covering the map with the minimal number of KH-polygons, then the covering that consists of larger KH-polygon must be selected.

From the foregoing discussions the following steps for obtaining simplified expression for f can be suggested:

1. Draw proposed KH-map for the function f of n variables. Start by covering with KH-polygons those 1 vertices that cannot be combined with any other 1 vertices.

2. Continue step 1 to those vertices that have only a single adjacent vertex for making KH-polygon of two vertices.

3. Next cover these TRUE vertices that yield KH-polygon of 2^k vertices and not part of any larger KH-polygon of 2^{k+i} vertices, where i > 0 and k = 2, 3, ... , n.

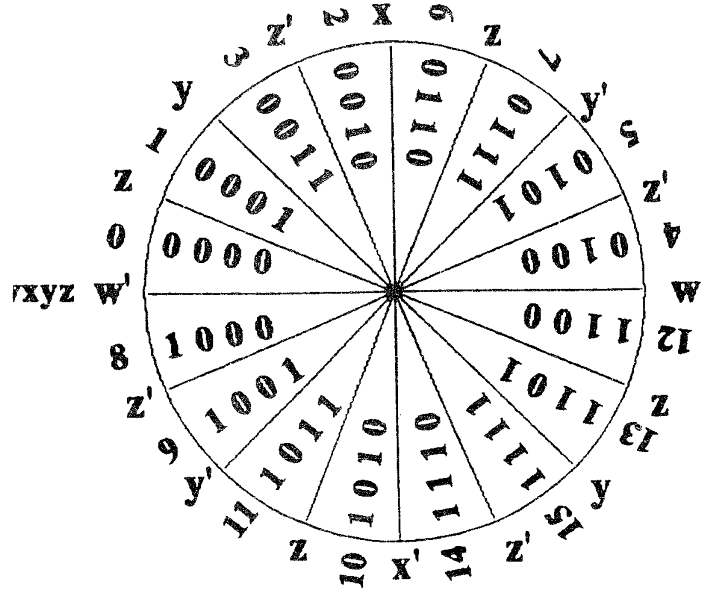
4. A minimal expression is one that corresponds to a collection of KH-polygons that are as large and as few as possible covering all TRUE vertices in the map of the function. A minterm for a KH-polygon of 2^k vertices, where 0<=k<=n, will contain n-k literal. Minterm for the KH-polygon will not contain those ith MSB, where ith MSB line bisects any arm of the KH-polygon.

EXAMPLES

Example 1. Fig. 3 represents KH-map for the expression $f(w,x,y,z) = \sum (0,1,3,5,7,8,9,10,13,15)$

For the polygon, abcd, ab and ad arms are bisected by w-line and z-line and minterm for the KH-polygon abcd will be x′y′ that contains only x and y variables. If one starts from any vertex and moves anti-clockwise then he will get for the first time negative y-line and x-line and hence y′ and x′ points. Minterm x′y′ can also be found from any vertex of KH-polygon eliminating appropriate variable(s). In similar way expression for KH-polygon abef is x′z′ and for ghij, it is xz.

Figure 2: KH-map for 4 variables



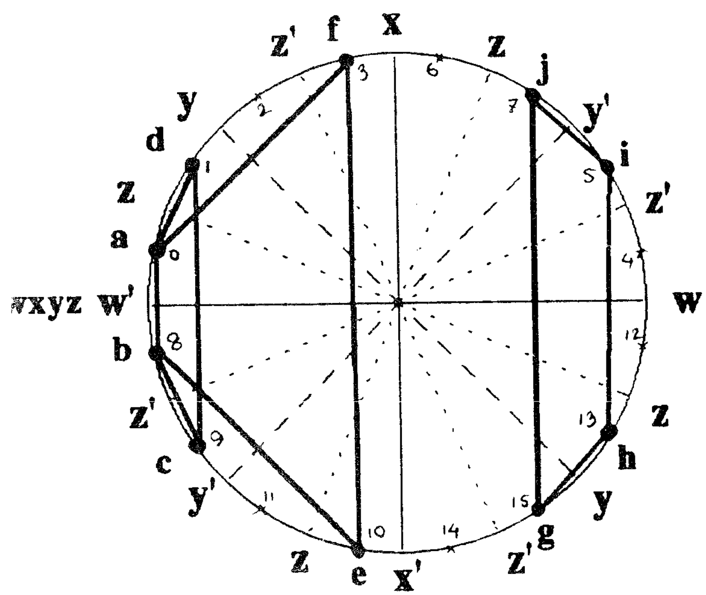
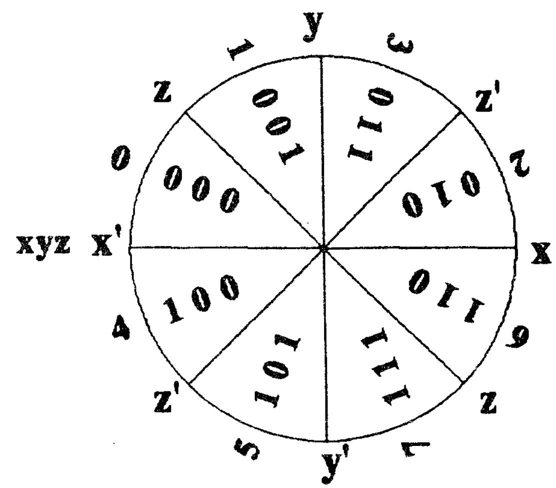


Figure 3: Realization of a 4-variable function

Figure 1: KH-map for 3 variables



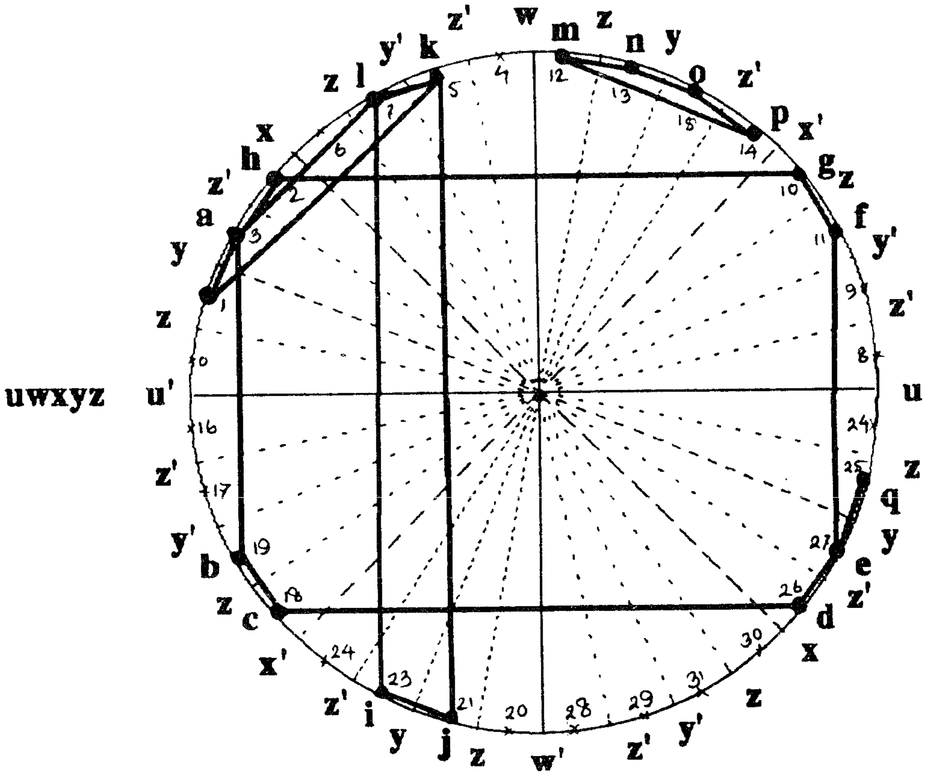


Figure 4: Realization of a 5-variable function

Hence, f(v,x,y,z) = x'y' + xz + x'z'.

Example 2. Fig 4 represents the expression f(u,w,x,y,z) = $\sum$(1,2,3,5,7,10,11,12,13,14,15,18,19,21,23,25,26,27) Arms ab, bc, and cd of KH-polygon abcdefgh have been bisected by u-line, z-line and w-line respectively and minterm is x'y may come from vertex u'w'x'yx'(vertex 2) or tracing end point of MSB line. Combining expressions for all polygon the function can be evaluated as f(u,v,x,y,z)= x'y(abcdefgh) + u'w'z(alkr) + w'xz(ijkl)+ u'wx(mnop) + uwx'z(eq). Content within parentheses represents corresponding polygon.

IV. COMPOUND KH-MAP

Concatenation is a very powerful feature of KH-map method. This process combines 2^m small KH-maps each of n variables. The arrangement is such that all KH-maps have common center and equal distance between two consecutive KH-map [Fig. 5]. Combined map will generate minimal expression covering m + n variables. This mapping will be termed as compound KH-map of m + n variables. The combination of the variable can be obtained from the following compound KH-algorithm:

COMPOUND KH-ALGORITHM

1. Let M_1, M_2, ... , M_2m indicates KH-maps from the innermost side and M_iC_j indicate jth cell in ith KH-map where, $0< i <= 2^m$ and $0< j <= 2^n$.

2. Initialize M_1 using KH-simple algorithm

3. set $p = 1, l = 2^n$

```
    for i= 1 to m do
        for j = 1 to p do
            for k = 1 to 2^n do
                M_{p+j}C_k = M_{p-j+1}C_k + l
            end-for
        end-for
            p = 2*p
            l = 2*l
    end-for
```

An alternative approach is to add 2^{k+n} to the content of each KH-map obtained from mirror image of 2^k KH-map as filled before, to fill next 2^k KH-map.

MINIMIZATION OF COMPOUND KH-MAP

Minimization technique in compound KH-map is almost similar to that of simple KH-map. In addition followings may provide better systematic way:

1. Solve each component KH-map of compound KH-map applying the rules as described in simple KH-map.

2. If two KH-polygon P1 and P2 in KH-map M1 and M2 are of same size and for each vertex in P1 P2 contains its adjacent vertex then P1 and P2 can be combined to form larger polygon.

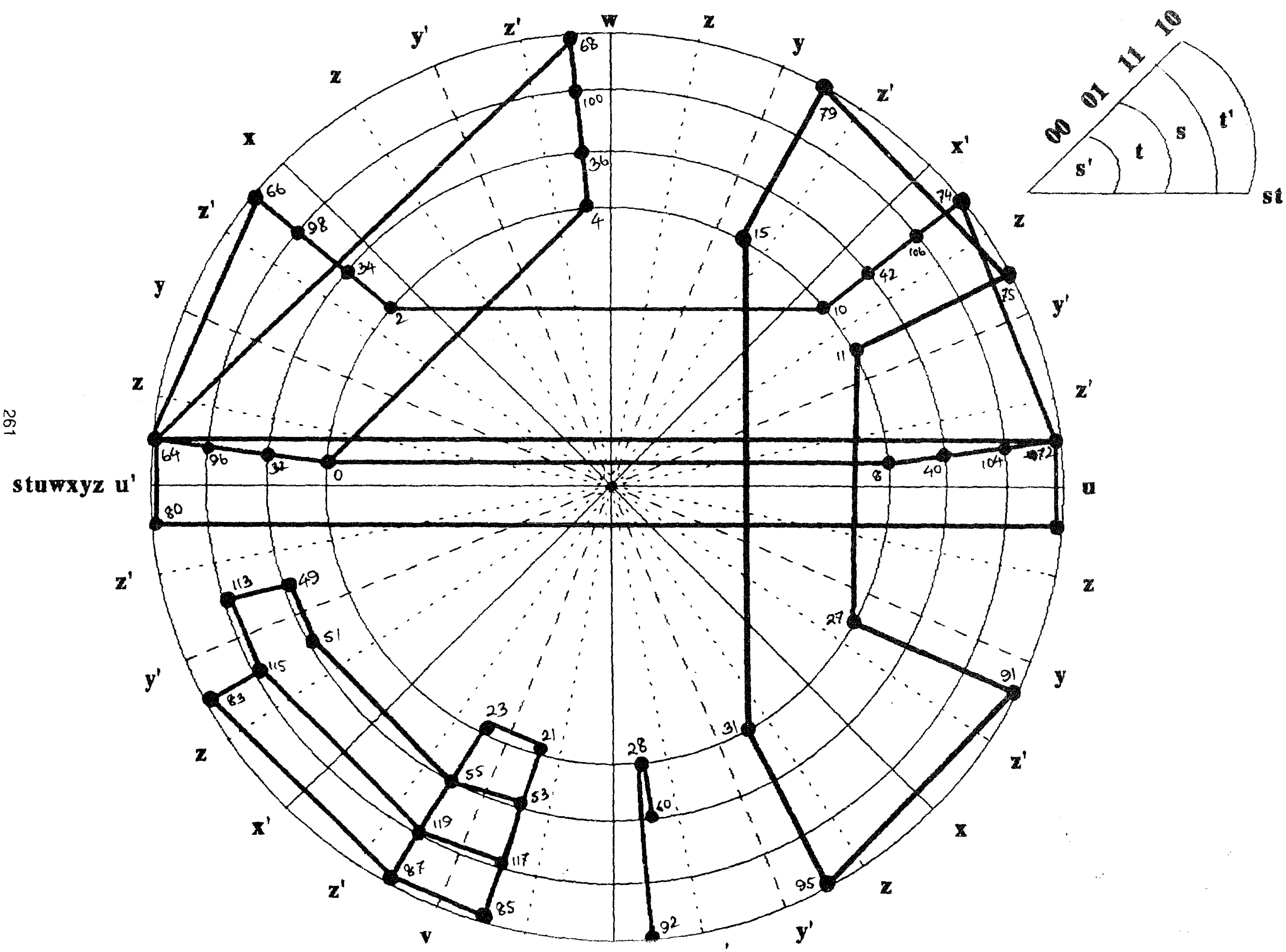
EXAMPLE

Fig. 5 represents the KH-map of the function f(s,t,u,w,x,y,z) = $\sum$ (0, 2, 4, 8, 10, 11, 15, 21, 23, 27, 28, 31, 32, 34, 36, 40, 42, 49, 51, 53, 55, 60, 64, 66, 68, 72, 74, 75, 79, 80, 83, 85, 87, 88, 91, 95, 92, 96, 98, 100, 104, 106, 109, 113, 115, 117, 119).

From the KH-map the expression can be evaluated as

f(s,t,u,w,x,y,z) = u'x'z' + t'wyz + u'w'y'z' + uw'xz + tuw'z + suw'yz + st'x'ty'z' + t'uwxy'z' + s'uwxy'z' + stu'wxy'z.

Figure 5: Compound KH-map for 7 variables



V. CONCLUSION

Proposed compound KH-map can be extended to large number of variables. This map provides a regular geometric structure that is more intuitive than other methods having non-geometric pattern. Using large compound KH-maps and, a switching function with a large number of variable can be simplified.

REFERENCES

[1].Bartee, T.C.:Computer Design of Multiple Output Logical Networks, IRE Trans. Electron. Computers, vol. EC-10, no. 1, pp. 21-30, March, 1961.

[2]. K. C. Chen and M. Fujita, "Efficient Sum-To-One Subset Algorithm for Logic Optimization," Proc. DAC, 1992.

[3]. A. Ghosh, S. Devadas and A. R. Newton "Heuristic Minimization of Boolean Relation Using Testing Technique," Proc. CICC, 1990.

[4]. Karnaugh, M. : The Map Method for Synthesis of Combinational Logic Circuits, Trans. AIEE. pt. I, vol. 72, no. 9, pp.593-599, 1953.

[5. McCluskey, E. J.,Jr. :Minimization of boolean functions, BellSystem Tech. J., vol. 35, no 6, pp. 1417-1444, Nov. 1956.

[6]. McCluskey, E.J., and H. Schorr: Essential Multiple-Output Prime Implicants, in Mathematical Theory of Automata, Proc. Polytech. Inst. Brooklyn Symp.,vol. 12, pp. 437-457, April, 1962.

[7]. M. Perkowski, P. Wu, and K. Pirkl. Kuai-exact: a new approach for multivalued logic minimization in VLSI synthesis. In IEEE International Symposium on Circuits and Systems, 1989.

[8]. Quine, W. V. : The Problem of Simplifying Truth Function, Am. Math. Monthly, vol. 59, no. 8, pp. 521-531, October, 1952.

[9]. Quine, W. V. : A Way to Simplifying Truth Functions Am. Math. Monthly, vol. 62, no. 9, pp. 627-631, November, 1955.

[10]. Veitch E. W. : A Chart Method for Simplifying Truth Functions, Proc. ACM, Pittsburgh, May 2-3, 1952, pp. 127-133.

[11. A.T.M. Shafiqul Khalid : A New Mapping Method for Boolean Function Manipulation, has been accepted for publication in the journal of Institute of Engineers Bangladesh(IEB).

[12]. A.T.M. S. Khalid : A New Mapping Technique for Simplifying Switching Expressions, R-2, CSE, Bangladesh University of Engineering & Technology, 1994.